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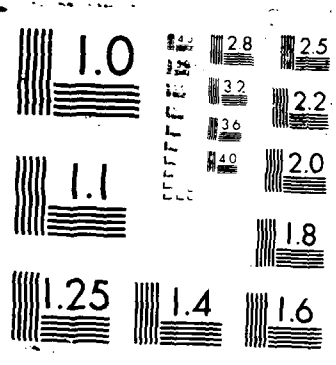
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The top Lyapunov exponent of a system of stochastic differential equations was investigated. Brownian motion paths on a Riemannian manifold were discussed and several theoretical results were obtained. A new asymptotic formula for the volume of a small extrinsic ball in a submanifold was obtained. Finally, an invariance principle for Lie groups was obtained.

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Submitted by Mark A. Pinsky, Professor of Mathematics,  
Northwestern University

Our work on this grant consisted of several projects which will be discussed separately. Some of the work was carried out in collaboration with other authors, who will be cited when appropriate. The list of references at the end documents the work which will eventually appear in research journals.

1. Lyapunov exponents of nilpotent systems with noise

In joint work with V. Wihstutz, we have investigated the top Lyapunov exponent of a system of stochastic differential equations of the form  $dx = Ax dt + \sigma Bx dw$  where  $w$  is a standard Wiener process with Stratonovich multiplication and  $A$  is a  $d \times d$  matrix with  $A^d = 0$  but  $A^{d-1}$  non-zero. We have analyzed the top Lyapunov exponent, which is defined as  $\lambda_\sigma = \lim_{t \rightarrow \infty} t^{-1} \log |x(t)|$ . It is known that under weak non-degeneracy conditions  $\lambda_\sigma$  is independent of  $(w, x_0)$ . We study the behaviour of  $\lambda_\sigma$  in the limiting cases  $\sigma \rightarrow 0$  and  $\sigma \rightarrow \infty$ . In the first case we obtain an asymptotic expansion of the form  $\lambda_\sigma = \sum \lambda_j \sigma^{j\alpha}$  ( $\sigma \downarrow 0$ ) where the exponent  $\alpha = 2/(2r-1)$  and  $\lambda_j = 1, \dots, r$  for a mild non-degeneracy condition on the pair  $(A, B)$ . The asymptotic formula reduces to a one-term formula

for a canonical choice of  $(A,B)$ . In the case of large noise it is natural to interchange the roles of  $A$  and  $B$ , obtaining similar asymptotic formulas for non-degenerate  $(A,B)$  and an exact one-term formula for a canonical choice of  $(A,B)$ . We present a number of applications to random Schrodinger equations, random harmonic oscillators and particles in a one-dimensional white noise potential. The results so obtained constitute a generalization and simplification of the well-known work of Auslender and Muhlstein who studied the Lyapunov exponent of stochastic systems with white noise coefficients. Our method depends solely on perturbation theory and thus avoids the cumbersome procedure of expanding a multiple integral into an asymptotic series in a small parameter which occurs in the denominator of the exponent of the integrand. The prototype problem of this type is the second order equation  $x'' = \sigma x w'$  which models a free-particle in one dimension under the influence of multiplicative white noise. The physical counterpart is that of a pendulum in outer space, where the effects of gravity are absent but the stochastic forces are operative.

## 2. Stochastic parallel translation of conditioned motion

In joint work with Ming Liao, we have discussed Brownian motion paths on a Riemannian manifold and the associated process of "stochastic parallel displacement". If we condition the Brownian motion to start at a point  $m \in M$  and end at the point  $\exp_m rz$  where  $z \in M_m$  and  $r > 0$ , we may compute  $E\xi$ , the average of the parallel translate of the vector  $\xi \in M_m$ . In order to determine the effect of curvature we obtain an asymptotic expansion of  $E\xi$  for small  $r > 0$ . As a consequence it is proved that  $E\xi$  lies in the radial direction provided that the sectional curvatures of  $M$  are constant. Conversely, we prove that in case  $\dim M = 2$ , this condition is also necessary: if  $E\xi$  lies in the radial direction, then  $M$  has constant Gaussian curvature. The method of proof depends on an extension of the perturbation theory which was previously developed to study the harmonic measure of a small sphere in a Riemannian manifold.

### 3. Volume of a small extrinsic ball in a submanifold

In joint work with Leon Karp, we have dealt with asymptotic results in differential geometry in which curvature invariants appear as coefficients in the expansions. Historically the first such formulas were due to Bertrand, Diguët and Puiseux who expressed the area of a non-Euclidean disk in terms of the curvature, to first order when the radius of the disk tends to zero. This formula was later generalized to geodesic balls in arbitrary Riemannian manifolds by Cartan with subsequent improvements by Gray and VanHecke. These papers yield characterizations of Euclidean space and other model spaces in terms of the volume of small geodesic balls. In order to obtain more effective characterizations of Euclidean space and other model spaces, several papers have been devoted to the mean exit time of Brownian motion. The asymptotic formula for the mean exit time involves new quadratic curvature invariants. In the case of the mean exit time from a tubular neighborhood, one obtains a quadratic form in the principal curvatures whose lowest eigenvalue is simple and corresponds to umbilic submanifolds. In the case of extrinsic balls of an immersed manifold, the mean exit time is expressed in terms of the mean curvature, to the first order of asymptotics. This leads to a characterization of minimal hypersurfaces in terms of the mean exit time of Brownian motion. In our work we consider a submanifold of  $R^n$  and its intersection with a small ball of  $R^n$ . The volume of the resulting intersection is computed in the asymptotic limit of radius tending to zero. This leads to a new asymptotic formula for the volume, in which appears a quadratic form which generalizes that obtained in studying the mean exit time of Brownian motion from a tubular neighborhood of a hypersurface. Finally we obtain some characterizations of spheres and totally geodesic imbeddings by means of the volume.

4. Invariance principle for Lie groups. In research supported by this grant, Dr. J. Watkins has studied the central limit theorem for Lie groups under a mixing hypothesis. The background for this problem lies in the early work of M. Donsker on the invariance principle for sums of independent and identically distributed real-valued random variables. Donsker's theorem asserts that if we endow the space of right-continuous paths with

the Skorohod topology, then the distribution of the normalized sum converges weakly to the Wiener measure when the number of summands becomes large. Donsker's theorem was extended by Billingsley to families of dependent random variables where the limiting random variable is not necessarily normal. The central limit theorem for this setting was proved jointly by Rosenblatt and Ibragimov in 1956. Billingsley proved the invariance principle for this case under a mixing hypothesis. Watkins' work was first carried out in the context of random variables taking values in the general linear group and later generalized to an arbitrary Lie group.

## REFERENCES

1. M. Pinsky and V. Wihstutz, Lyapunov exponents for nilpotent Ito Systems, preprint, June 1987
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3. L. Karp and M. Pinsky, Volume of an extrinsic ball in a submanifold, preprint, July 1987
4. J. Watkins, Donsker's invariance principle for Lie groups, preprint, June, 1987.



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